Predicting Corporate Financial Distress: A Time-Series CUSUM Methodology

EMEL KAHYA

School of Business
Rutgers University
Camden, NJ 08102

PANAYIOTIS THEODOSSIOU

School of Business
Rutgers University
Camden, NJ 08102

Panayiotis Theodossiou
School of Business
Rutgers University
Camden, NJ 08102

(609) 225-6594 (Office)
(609) 225-6632 (Fax)

*The authors thank the editor, an anonymous reviewer of the journal, Edward Altman, Haim Falk, Milton Leontiades, Teppo Martikainen and the participants of the faculty seminars at Hofstra University, Rutgers University, and the University of Cyprus for valuable comments and suggestions. The paper was presented at the 156th Annual Meeting of the American Statistical Association, August 1996, and the 3rd Annual Multinational Finance Conference, June 1996. The authors received financial support from the Research Council Fund, Rutgers University, June 1996.

Predicting Corporate Financial Distress: A Time-series CUSUM Methodology

Abstract

This paper develops a financial distress model using the statistical methodology of time-series Cumulative Sums (CUSUM). The model has the ability to distinguish between changes in the financial variables of a firm that are the result of serial correlation and changes that are the result of permanent shifts in the mean structure of the variables due to financial distress. Tests performed show that the CUSUM model is robust over time and outperforms other models based on the popular statistical methods of Linear Discriminant Analysis and Logit.

Key words: financial distress model, Linear Discriminant Analysis, Logit, time-series CUSUM, vector autoregressive

1. Introduction

Explanatory variables included in financial distress models exhibit strong positive serial correlation over time, e.g., Theodossiou (1993), and in many cases they are not stationary.[1] As such, positive deviations of these variables from their means in one period are generally followed by positive deviations in subsequent periods while negative deviations are followed by negative deviations. The presence of serial correlation may be attributed to active attempts by the management to align the variables with their population means and/or systematic micro- and macroeconomics effects operating on the firm, e.g., Lee and Wu (1988).

Under stationarity, the deviations of the variables for healthy firms are transitory; thus, over time the variables revert back to their means in the healthy population. The reversion time depends on the degree of serial correlation in the variables.[2] For financially distressed firms, the deviations of the variables also include a non-transitory component which is due to permanent shifts in the mean structure of the variables toward the failed population. These shifts are initially small in magnitude and become larger as the firms approach the point of economic collapse.

Past financial distress models based on Linear Discriminant Analysis (LDA), Logit, Probit, proportional hazard and other similar statistical models do not account for the time-series behavior of financial variables. Therefore, they cannot distinguish between transitory and non-transitory changes in a firm's financial variables. In addition, the first three models are static and assess the financial condition of a firm using data from a single period, ignoring important past information regarding the firm's financial performance.

This paper develops a financial distress model that accounts for the above time-series behavior of financial variables. The model is based on the statistical methodology of time-series CUSUM developed by Theodossiou (1993). The paper extends significantly the work of

Theodossiou (1993) by avoiding problems associated with non-stationary variables and the definition of financial distress. Moreover, the paper incorporates several refinements of the CUSUM model and focuses on the intuitive rather than the statistical aspects of the model.

The paper proceeds as follows: Section 2 presents the statistical methodology of time-series CUSUM as applied in the area of predicting business failures. Section 3 describes the sampling methodology and variables used. Section 4 deals with the identification, estimation, and evaluation of the forecasting performance of the CUSUM model. Section 5 presents robustness tests for the best CUSUM model. The paper ends with a summary and concluding remarks.

2. Time-series CUSUM methodology

Let $X_{i,t} = \left[X_{1,i,t}, X_{2,i,t}, \ldots, X_{p,i,t} \right]$ be a row vector of p attribute variables for the i^{th} firm at time t with predictive ability with respect to financial distress. The sequence of attribute vectors $X_{i,1}, X_{i,2}, \ldots, X_{i,t}, \ldots$ for a healthy firm is stationary and follows a "good" performance distribution with constant population mean over time.[3] For a financially distressed firm, the sequence of attribute vectors shifts (switches) gradually at some random time from a "good" performance distribution to a "bad" performance distribution. These shifts are initially small in magnitude and become larger as the firm approaches the point of economic collapse. A CUSUM model determines in an optimal manner the starting point of the shift and provides a signal of the firm's deteriorating condition as soon as possible after the shift.

2.1. *Time-series behavior of variables*

The time-series behavior of the attribute variables for healthy and failed firms can be adequately described by a finite order vector autoregressive model, VAR(k), as follows:

$$X_{i,t} = A_{f,s} + A_h + X_{i,t-1}B_1 + \cdots + X_{i,t-k}B_k + \varepsilon_{i,t}, \quad for\ s = 1,2,\ldots,m \tag{1a}$$

$$A_{f,s} = 0 \quad for\ healthy\ firms\ \ and\ s > m, \tag{1b}$$

$$E\left(\varepsilon_{i,t}\right) = 0, \quad E\left(\varepsilon_{i,t}'\varepsilon_{i,t}\right) = \Sigma, \quad and \quad E\left(\varepsilon_{i,t}'\varepsilon_{j,r}\right) = 0, \qquad for\ i \neq j\ and/or\ r \neq t, \tag{1c}$$

where $\varepsilon_{i,t} = \left[\varepsilon_{1,i,t}, \varepsilon_{2,i,t}, \ldots, \varepsilon_{p,i,t}\right]$ is an independently distributed error vector with mean zero and variance-covariance matrix equal to Σ, $A_h = \left[A_{1,h}, A_{2,h}, \ldots, A_{p,h}\right]$ is a vector of intercepts for healthy firms, $A_{f,s} = \left[A_{1,f,s}, A_{2,f,s}, \ldots, A_{p,f,s}\right]$ are deviations from A_h associated with attribute vectors for failed firms extracted s years prior to failure, and $B_1, B_2, \ldots, B_k$ are $p \times p$ matrices of VAR coefficients. The term $A_{f,s}$ captures permanent shifts in the mean structure of the variables due to financial distress. By construction, $A_{f,s}$ is equal to zero for all attribute vectors (observations) of the healthy firms. Also, $A_{f,s}$ is zero for observations of failed firms extracted prior to the starting point of the shift in the distribution of $X_{i,t}$ from the healthy population to the failed population (i.e., for $s > m$). Equation $E\left(\varepsilon_{i,t}'\varepsilon_{j,r}\right)$, for $i \neq j$ and/or $r \neq t$, implies that the error term is uncorrelated across firms and time. For practical purposes, the variance-covariance matrix of the error term Σ is specified to be equal in both groups, e.g., Marks and Dunn (1974) and Altman et al. (1977).

A necessary condition for the above VAR process to be stationary is that the roots of the polynomial $\det\left(I - B_1 z - \cdots - B_k z^k\right) = 0$ lie outside the complex unit circle, where det denotes the determinant, I is an identity matrix and z are the roots of the polynomial, e.g., Judge et al. (1985), pp. 656-659. Stationarity implies that the variables are mean-reverting in the sense that when they depart from their mean values they return to them in the near future. Stationarity of the attribute vectors $X_{i,t}$ also has significant implications for the robustness of financial distress models over time, e.g., Theodossiou and Kahya (1996).

Under stationarity of the VAR process, the unconditional mean of $X_{i,t}$ for healthy firms is equal to $\mu_h = A_h + \mu_h B_1 + \cdots + \mu_h B_k = A_h \left(I - B_1 - \cdots - B_k \right)^{-1}$. Substitution of the first formula into equation 1a gives

$$X_{i,t} - \mu_h = A_{f,s} + \left(X_{i,t-1} - \mu_h \right) B_1 + \cdots + \left(X_{i,t-k} - \mu_h \right) B_k + \varepsilon_{i,t}, \qquad for \ s = 1, 2, \ldots, m, \tag{2}$$

where $\left(X_{i,t} - \mu_h \right)$ denotes the deviations of the variables from their mean values in the healthy population for firm *i* at time *t*. These deviations are composed of the transitory component, which includes the autoregressive part and error term of the VAR model, and the non-transitory component $A_{f,s}$, which is due to permanent shifts in the mean structure of the variables toward the failed population. The above formulation is similar to that used in Theodossiou (1993).

2.2. *The CUSUM model*

Based on the sequential probability ratio tests and the theory of optimal stopping rules, Theodossiou (1993) shows that the CUSUM model will provide a signal of the firm's deteriorating condition as soon as:

$$C_{i,t} = \min\left(C_{i,t-1} + Z_{i,t} - K, \ 0 \right) < -L, \quad for \ K, L > 0 \tag{3}$$

where $C_{i,t}$ and $Z_{i,t}$ are a cumulative (dynamic) and an annual (static) time-series performance score for the i^{th} firm at time *t* and *K* and *L* are sensitivity parameters taking positive values.[4]

The score $Z_{i,t}$ is a complex function of the attribute variables $X_{i,t}$ accounting for serial correlation in the data. It is calculated using the formula:

$$Z_{i,t} = \beta_0 + \left(X_{i,t} - A_h - X_{i,t-1} B_1 - \cdots - X_{i,t-k} B_k \right) \beta_1 = \beta_0 + A_{f,s} \beta_1 + \varepsilon_{i,t} \beta_1, \tag{4}$$

$$\beta_0 = (1/2D) A_f \Sigma^{-1} A_f' = D/2, \tag{5}$$

$$\beta_1 = -(1/D) \Sigma^{-1} A_f', \quad \textit{and} \tag{6}$$

$$D^2 = A_f \Sigma^{-1} A_f', \tag{7}$$

where β_0 and β_1 are the CUSUM parameters and D is the Mahalanobis generalized distance of the error terms of the variables in the healthy and failed populations. Note that, for simplicity of notation, $A_f \equiv A_{f,1}$. As shown in the appendix, the annual performance score $Z_{i,t}$ has a positive mean of $D/2$ in the healthy population and a negative mean of $-D/2$ in the failed population, for s=1. Moreover, the $Z_{i,t}$ scores are serially uncorrelated over time and have a variance of one for both the healthy and failed firms.

According to the CUSUM model, the overall performance of a firm at time t is measured by the cumulative score $C_{i,t}$. For a typical healthy firm, the $Z_{i,t}$ scores are positive and greater than K, thus the $C_{i,t}$ scores are equal to zero. For a typical failing firm, the $Z_{i,t}$ scores fall below K, thus the $C_{i,t}$ scores accumulate negatively. A signal of the firm's changed condition is given at the first time $C_{i,t}$ falls below $-L$. Note that the $C_{i,t}$ scores would increase and go back to zero if and only if the firm displayed $Z_{i,t}$ scores greater than K.[5]

2.3 *Sensitivity parameters K and L*

The sensitivity parameters K and L determine the time between the occurrence and the detection of a change in the financial condition of a firm. The larger the value of K, the lower the probability of misclassifying a failing firm as healthy and the larger the probability of misclassifying a healthy firm as failed. The opposite is true with the parameter L.

Define:

$$P_f = prob\left(C_{i,t} > -L \mid firm\,is\ failed\ and\ s = 1\right), \quad and \tag{8a}$$

$$P_h = prob\left(C_{i,t} \leq -L \mid firm\,is\,healthy\right) \tag{8b}$$

to be respectively the percentages of failed and healthy firms in the population not classified correctly by the CUSUM model. These are also known as Type I and Type II errors and they are functions of the parameters *K* and *L*. The optimal values of *K* and *L* are derived by solving the dynamic optimization problem:

$$\min_{K,L} EC = w_f\, P_f\,(K, L) + (1 - w_f\,) P_h(K, L), \tag{9}$$

where w_f and $w_h = 1 - w_f$ are investors' specific weights attached to the error rates P_f and P_h. The *EC* is specified as a function of P_f because the CUSUM model is developed for the purpose of predicting a shift in the mean of a firm's attribute vector from μ_h to $\mu_f \equiv \mu_{f,1}$, but not necessarily to any intermediate state.

The weights $w_f = \pi_f c_f / (\pi_f c_f + \pi_h c_h)$ *and* $w_h = \pi_h c_h / (\pi_f c_f + \pi_h c_h)$ are functions of the a-priori probabilities π_f and $\pi_h = 1 - \pi_f$, measuring the actual proportion of failed and healthy firms in the population, and the costs c_f and c_h associated with the misclassification of failed and healthy firms. In the absence of specific weighs, the choice of equal weights ($w_f = w_h = \frac{1}{2}$) appears to be a reasonable alternative. This is because, π_f is generally smaller than π_h, but c_f is generally greater than c_h. The *EC* criterion with equal weights is used within a neural network framework to select the profile of variables with the best overall forecasting performance. The error rates for various combinations of the weights used in the optimization of the above function are calculated using the jackknife method described in section 4.4.

3. Sampling and financial variables

3.1. *Sampling methodology*

The selection of the sample of financially distressed (failed) firms is based on debt default criteria, such as debt default or debt renegotiation attempts with creditors and financial institutions. Information on debt default and debt renegotiation is gathered from various annual issues of the Wall Street Journal Index (WSJI). The time of failure is chosen as the first time the firm experienced one of the signs of failure. The above definition of financial distress avoids many of the problems associated with the legal definition of business failure.

Specifically, the 1978 federal Bankruptcy Code made it easy for firms to file petitions for Chapter 7 liquidation or Chapter 11 reorganization. As a result, many firms filed for bankruptcy liquidation or reorganization for reasons other than financial distress. For example, in 1982, the Manville Corp. filed under Chapter 11 as a way of dealing with lawsuits from individuals claiming exposure to its asbestos products. In 1987, Texaco filed under Chapter 11 to reduce its liability to Pennzoil. In 1994, Petrie Stores Corp. received a favorable ruling from the IRS, allowing a tax-free liquidation. None of these companies exhibited any signs of financial distress prior to filing for bankruptcy. On the other hand, many financially distressed firms never file for bankruptcy because of acquisition. For example, in 1980, American Motors Corp. (AMC) was rescued by Renault while experiencing serious debt-servicing problems. In 1987, AMC was acquired by the Chrysler Corp. Similarly, in 1986, Clevepak Corp. was acquired by the Madison Management Group, Inc., five months after suspending payment of principal on debt.

These examples show that the legal definition of failure results in "contaminated" healthy and failed samples. That is, the failed sample will include firms that filed for bankruptcy for reasons other than financial distress and will disregard financially distressed firms that never filed for bankruptcy. The latter firms may be included in the healthy sample. Moreover, many financially distressed firms file for bankruptcy and operate under a reorganization plan for several years before filing for bankruptcy liquidation. This makes the determination of the timing of failure and collection of data a problematic one. The use of contaminated samples and incorrect information on the timing of failure distorts the distributional properties of the financial variables in the sample and impairs the forecasting ability of the models.

The samples obtained using the debt default criteria includes 117 healthy firms and 72 failed firms. Data for the firms are extracted from the 1993 annual industrial and research COMPUSTAT tapes and span the period 1974–91. The sample of healthy firms is compiled from a sample of 150 firms collected randomly from the population of about 1,000 manufacturing and retailing firms listed on the NYSE and the AMEX in 1992. Note that this sample is large enough to provide a good coverage of the population. Twenty-two of the firms are dropped from the sample because of non-continuous data and/or a few annual observations. The remaining 128 firms are thoroughly screened for signs of financial distress using the annual volumes of the WSJI for the period 1978-1995 (latest volume). Eleven of these firms are found to exhibit signs of financial distress; thus, they are classified as failed. The remaining failed firms are identified using debt default criteria from a population of about 300 manufacturing and retailing firms delisted from the NYSE and AMEX during the period 1982-92 because of bankruptcy liquidation, bankruptcy reorganization, privatization, merger, and acquisition. OTC firms are not considered because they are generally smaller than NYSE and AMEX firms and, as such, their financial attributes with respect to bankruptcy are expected to be different, e.g., Edmister (1972). Moreover, petroleum (SIC=2911) and mining firms (SIC=3312, 3330 and 3334) are not considered because they possess financial attributes that are statistically different from those of other manufacturing firms.

3.2. *Financial variables*

The variables considered are mostly derived from the broad class of financial ratios found to be significant explanatory variables in past financial distress models. Table 1 provides a list of the variables, the formulas used to compute their values, and citations for a sample of studies that considered the variables. The variables are classified into the categories of liquidity, profitability, financial leverage, size, and other variables. In addition to the levels, the paper considers first differences (changes) in the variables over time. First differences provide useful information regarding financial distress. Moreover, they are preferable to variables' levels because levels are generally non-stationary over time.

4. CUSUM model development

4.1. *Model identification*

The identification of the best CUSUM model is done by means of a neural network search procedure based on the *EC* criterion; that is, by choosing the profile of explanatory variables that minimizes the model's expected cost function given by equation 9. This profile of explanatory variables is chosen from a set of 54 variables which includes the 27 variables listed in table 1 and their first differences. All models considered are tested for stationarity over time. Non-stationary models are dropped. Interestingly, most of the popular financial variables included in past financial distress models produce non-stationary models with deteriorating forecasting performance over time.

The set of 54 variables generates an extremely large number of profiles of financial variables.[6] Searching all possible profiles is not desirable. For practical purposes, the search procedure is programmed to allow for one explanatory variable from each major category of variables to enter a model at a time. The latter approach is reasonable, because the inclusion of two or more variables from the same category is not expected to improve significantly a model's forecasting performance.

The best stationary CUSUM model produced by the search procedure includes four explanatory variables. These are the change in the logarithm of deflated total assets, the change in the ratio of inventory to sales, the change in the ratio of fixed assets to total assets, and the change in the ratio of operating income to sales. The above model exhibits at least as good an average performance over time as the best non-stationary model.

Figure 1 illustrates the annual sample means and standard deviations of the four variables for the sample of 117 healthy firms. The standard deviations for the variables are presented in the form of plus and minus two standard deviations from the means. As such, they provide a distributional range for 95 percent of their values. The straight line gives the overall mean of the variables for all healthy firms and years. The results indicate that all four variables are relatively stable over time.

Figure 2 illustrates the means and standard deviations of the four variables by year prior to failure for the sample of 72 failed firms. The straight line gives the overall mean of the variables for the 117 healthy firms. The means of the variables in the failed sample are lower for the change in the logarithm of deflated total assets, the change in the ratio of fixed assets to total assets, and the change in the ratio of operating income to sales, and higher for the change in the ratio of inventory to sales. These means, at one year prior to failure (s=1), are statistically different from their respective overall means in the healthy sample, except for the mean of the change in the ratio of fixed assets to total assets. The latter variable, however, in combination with the other three variables, improves the predictive ability of the model.

4.2. *VAR estimates for explanatory variables*

The VAR estimates for the four explanatory variables are obtained by fitting equation 1a to the data for the 72 failed firms and 117 healthy firms over the period 1974–91. Pooling of the data in the estimation is necessary because of the small number of yearly observations for each firm as well as homogeneity reasons. In the best case, 18 yearly observations are available while, on many occasions, firms had a few yearly observations. The VAR estimates are obtained by maximizing the log-likelihood function of the pooled sample, e.g., Johansen (1995), p. 18. Due to random sampling, the log-likelihood function is specified as the sum of individual firm log-likelihood functions.

The identification of the order of the VAR model is performed using the Akaike's information criterion; that is, by minimizing $AIC = \ln\left(\det \tilde{\Sigma}\right) + 2\,M/NT$, where M represents the number of estimated VAR coefficients, NT represents the number of annual observations for all firms in the pooled sample, and $\tilde{\Sigma}$ is the estimate of the error covariance matrix based on the residuals of the pooled sample, denoted by $\tilde{\varepsilon}_{i,t}$. Specifically, $\tilde{\Sigma} = \sum \tilde{\varepsilon}'_{i,t}\tilde{\varepsilon}_{i,t} \big/ (NT - 5)$. The analysis of the data by means of AIC gives a first-order VAR model. It is important to note that the estimation and identification of the order of the VAR model are performed automatically by the neural network procedure (described previously).

The estimated VAR model is as follows:

$$X_{i,t} = \tilde{A}_f + \tilde{A}_h + X_{i,t-1}\tilde{B}_1 + \tilde{\varepsilon}_{i,t}\,, \tag{10}$$

where

$$X_{i,t} = \left[\Delta \ln(\text{Total assets})_{i,t} \quad \Delta\left(\frac{\text{Inventory}}{\text{Sales}}\right)_{i,t} \quad \Delta\left(\frac{\text{Fixed assets}}{\text{Total assets}}\right)_{i,t} \quad \Delta\left(\frac{\text{Operating income}}{\text{Sales}}\right)_{i,t} \right],$$

$$\tilde{A}_h = 10^{-2}\begin{bmatrix} 5.2824 & -.3218 & -.0088 & .0717 \\ (13.7) & (-4.69) & (-.99)* & (.84)* \end{bmatrix},$$

$$\tilde{A}_f = 10^{-2}\begin{bmatrix} -12.0413 & .1391 & -.2003 & -1.0674 \\ (-6.26) & (.38)* & (-.42)* & (-2.52) \end{bmatrix},$$

$$\tilde{B}_1 = \begin{bmatrix} \begin{matrix}.2863\\(14.9)\end{matrix} & \begin{matrix}0\\(0)\end{matrix} & \begin{matrix}.0171\\(3.79)\end{matrix} & \begin{matrix}-.0166\\(-3.92)\end{matrix} \\ \\ \begin{matrix}-.3727\\(-3.59)\end{matrix} & \begin{matrix}-.2104\\(-11.2)\end{matrix} & \begin{matrix}0\\(0)\end{matrix} & \begin{matrix}-.0817\\(-3.58)\end{matrix} \\ \\ \begin{matrix}.1783\\(2.25)\end{matrix} & \begin{matrix}0\\(0)\end{matrix} & \begin{matrix}0\\(0)\end{matrix} & \begin{matrix}0\\(0)\end{matrix} \\ \\ \begin{matrix}.3417\\(3.91)\end{matrix} & \begin{matrix}0\\(0)\end{matrix} & \begin{matrix}0\\(0)\end{matrix} & \begin{matrix}-.1907\\(-10.0)\end{matrix} \end{bmatrix},$$

ln denotes the natural logarithm and Δ denotes the first difference operator. Parentheses include the t-values of the estimates. Estimates of $\tilde{A}_{f,s}$, for $s = 2,\ldots,m$, are available upon request.

The VAR coefficients $\tilde{B}_1$ provide information on how the variables relate to their past values as well as to past values of the other variables. Statistically insignificant autoregressive coefficients are set equal to zero. In this respect, each equation is re-estimated using only past values for the variables that exert a statistically significant relationship on current values of each variable.

The pooled variance–covariance matrix in the healthy and failed samples (at one year prior to failure) is estimated from the residuals using the formula:

$$\tilde{\Sigma}_p = \frac{(N_h - 5)\tilde{\Sigma}_h + \left(N_f - 5\right)\tilde{\Sigma}_f}{N_h + N_f - 10}, \tag{11}$$

where N_h =1,958 is the total number of yearly observations for the 117 healthy firms, $\tilde{\Sigma}_h = \sum \tilde{\varepsilon}_{i,t}' \tilde{\varepsilon}_{i,t} / \left(N_h - 5\right)$ is 4x4 variance-covariance matrix of $\tilde{\varepsilon}_{i,t}$ in the healthy sample, N_f =71 is the number of observations extracted at one year prior to failure and $\tilde{\Sigma}_f = \sum \tilde{\varepsilon}_{i,t}' \tilde{\varepsilon}_{i,t} \big/ \left(N_f - 5\right)$ is 4x4 variance-covariance matrix of $\tilde{\varepsilon}_{i,t}$ in the failed sample using the residuals at one year prior to failure.[7]

$$\hat{\Sigma}_p = 10^{-2} \begin{bmatrix} 2.1916 & .1146 & -.0978 & .0460 \\ .1146 & .0666 & .0021 & -.0143 \\ -.0978 & .0021 & .1319 & -.0148 \\ .0460 & -.0143 & -.0148 & .0733 \end{bmatrix}$$

The pooled variance-covariance is the proper measure to use in equations 5-7, because the CUSUM model is developed for the purpose of predicting a shift in the mean of a firm's attribute vector from μ_h to μ_f, but not to any intermediate state, e.g., Amemiya (1981), p.1509.

4.3. *Estimation of the CUSUM model*

Substitution of $\tilde{A}_h, \tilde{A}_f, \tilde{B}_1$ and $\tilde{\Sigma}_p$ into equations 5–7 yields:

$\tilde{\beta}_0 = 0.4694,$

$\tilde{\beta}_1 = \begin{bmatrix} 6.5815 & -11.4976 & 7.8873 & 10.7195 \end{bmatrix}'$, and

$\tilde{D} = 0.9387.$

The estimated parameters for β_0, β_1, A_h and B_1 and equation 4 are used to calculate a firm $Z_{i,t}$ scores as follows:

$$Z_{i,t} = \tilde{\beta}_0 + \left(X_{i,t} - \tilde{A}_h - X_{i,t-1}\tilde{B}_1\right)\tilde{\beta}_1 . \qquad (12)$$

The CUSUM coefficients $\tilde{\beta}_1$ measure the impact of the variables on the firm's annual performance score $Z_{i,t}$ and provide an economic understanding of the variables as predictors of financial distress. The coefficients for the annual changes in the natural logarithm of deflated total assets, ratio of fixed assets to total assets, and ratio of operating income to sales have positive signs implying a positive marginal relationship between the variables and the firm's performance score $Z_{i,t}$. On the other hand, the coefficient for the change in the ratio of inventory to sales has a negative sign implying a negative marginal relationship. These results are easily justified on financial and economic grounds.

Specifically, the logarithm of deflated total assets is used as a proxy of the firm's size. Positive changes in this variable are indicative of positive annual growth rates for the firm. Healthy firms experience positive growth rates, while failing firms initially experience below average growth rates which become negative a few years prior to failure. Thus, negative growth rates of deflated assets are indicative of financial distress.

Firms experiencing problems promoting and servicing their products are expected to possess a larger level of inventory relative to their sales over time. This ratio is used as a proxy for management efficiency, e.g., Theodossiou et al. (1996). Positive changes in the ratio are indicative of management problems and thus relate negatively to financial distress.

Net fixed assets (property, plant, and equipment) are mainly used by firms to produce and distribute goods and services. Financially distressed firms frequently sell fixed assets to improve their liquidity position. On the other hand, healthy firms increase their fixed asset position by expanding or modernizing their plants. Therefore, decreases in this ratio are likely to be associated with deteriorating financial performance for the firm.

Finally, the ratio of operating income to sales is used as a proxy for the profitability of the firm. Positive changes in this ratio indicate improvements in the profitability and vice versa. Therefore, decreases in the ratio are associated with deteriorating financial performance.

Figure 3 illustrates the time path of the mean of $Z_{i,t}$ scores for failed firms in the sample starting from six years prior to failure to one year prior to failure. The horizontal lines at

$\tilde{D}/2$ and $-\tilde{D}/2$ denote the means of $Z_{i,t}$ in the healthy and failed (for s=1) samples, respectively. Note that the average scores for failed firms at six years prior to failure are close to the mean in the healthy sample. As the financial condition of the firms deteriorates, they move toward the failed sample mean of $-\tilde{D}/2$.

Interestingly, all explanatory variables included in the CUSUM model are expressed in first difference form (changes in the levels of the variables) over time. Note that the CUSUM scores $C_{i,t}$ for each firm are calculated recursively using the formula

$$C_{i,t} = \min\left(C_{i,t-1} + Z_{i,t} - .0587,\ 0\right) < -.8214 \tag{13}$$

Adverse changes in the levels of the four variables have a negative impact on the firm's performance score $Z_{i,t}$ causing it fall below the threshold K=.0587. Persistence of these adverse changes causes the CUSUM score $C_{i,t}$ to accumulate negatively over time, signaling the firm's deteriorating condition as soon as $C_{i,t}$ falls below $-L= -.8214$ (details on the determination of the optimal values of K and L are presented below). It can be easily shown that the $C_{i,t}$ score is a function of the levels of the variables, expressed in deviation form from their respective means in the healthy sample. The levels of the variables relate to $C_{i,t}$ in the same way their changes relate to $Z_{i,t}$.

4.4. *Determination of the optimal values of K and L*

The *EC* criterion is used to determine the optimal sensitivity parameters of the CUSUM model and evaluate its forecasting performance. As a first step in applying the *EC* criterion, the error rates of each estimated CUSUM model P_f and P_h are computed using the jackknife method with 250 replications.[8] During each replication, one healthy and one failed firm are randomly dropped from the data and all CUSUM parameters are re-estimated. Equation 3 is then used to calculate the CUSUM scores over time of the held-back firms for 1,600 combinations of K and L spanning uniformly the intervals [0, D/2] and [0, 5D], respectively. Next, all yearly observations for the held-back healthy firm and the observation at one year prior to failure (s=1) for the held-back failed firm are reclassified using their respective CUSUM scores. A tally of the number of misclassified observations is kept for each combination of K and L. P_h is computed by dividing the number of misclassified observations by the total number of observations of all 250 held-back healthy firms. P_f is computed by dividing the number of misclassified failed observations at one year prior to failure by 250.

Equation 9 is then used to compute the model's expected cost function EC for values of w_f ranging between .4 and .6 with increments of .05 and all 1,600 combinations of K and L. For each value of w_f, the K and L combination that minimizes EC is chosen. The optimal combinations of K and L and error rates of the CUSUM model in the healthy and failed samples for a given value of w_f are presented in panel A of table 2. Note that the last three columns of the panel present the error rates in the failed sample using the CUSUM scores corresponding to two, three, and four years prior to failure. These error rates are denoted by $P_{f,2}$, $P_{f,3}$ and $P_{f,4}$, respectively.

Note that for $w_f = w_h = \frac{1}{2}$, the optimal parameters of the CUSUM model are K = .0587 and L = .8214. The model's error rate in the healthy sample is $P_h = 17.06$ percent and in the failed sample, using data from one year prior to failure (s = 1), is $P_f = 18.31$ percent. The model's expected cost for s = 1 is EC = 17.69. The respective error rates in the failed sample using data from two, three, and four years prior to failure are $P_{f,2} = 40.28$ percent, $P_{f,3} = 45.83$ percent, and $P_{f,4} = 60.56$ percent. As expected, these error rates increase because it becomes harder to predict financial distress further back from the point of failure.

4.5 *CUSUM vs. LDA and Logit models*

In the absence of serial correlation in the data $B_1 = 0, A_h = \mu_h, A_h + A_f = \mu_f, A_f = \mu_h - \mu_f$ and the CUSUM equations 4–6 reduce to those of LDA. That is,

$$Z_{i,t} = \beta_0 + \left(X_{i,t} - \mu_h\right)\beta_1 = \beta_1^* + X_{i,t}\beta_{1,} \tag{14}$$

$$\beta_0^* \equiv \beta_0 - \mu_h\beta_1 = (1/2D)\left(\mu_h - \mu_f\right)\Sigma^{-1}\left(\mu_h + \mu_f\right)', \tag{15}$$

$$\beta_1 = -(1/D)\left(\mu_h - \mu_f\right)', \quad \textit{and} \tag{16}$$

$$D^2 = \left(\mu_h - \mu_f\right)\Sigma^{-1}\left(\mu_h - \mu_f\right)', \tag{17}$$

where μ_h is the mean of $X_{i,t}$ in the healthy sample, μ_f is the mean of $X_{i,t}$ in the failed sample using data at one year prior to failure, Σ is the pooled variance-covariance matrix of the variables, β_0^* and β_1 are the LDA coefficients, and D is the Mahalanobis generalized distance. The LDA estimates below are obtained in the conventional way, e.g Amemiya (1991), p. 1509, for the details,

$$\tilde{\mu}_h = 10^{-2}\left[7.3852 \quad -.2542 \quad .1078 \quad -.0153\right],$$

$$\tilde{\mu}_f = 10^{-2}\left[-7.082 \quad -.2157 \quad -.2078 \quad -.7754\right],$$

$$\tilde{\Sigma}_p = 10^{-2}\begin{bmatrix} 2.3839 & .1163 & -.0879 & .0371 \\ .1163 & .0717 & .0014 & -.0132 \\ -.0879 & .0014 & .1325 & -.0156 \\ .0371 & -.0132 & -.0156 & .076 \end{bmatrix},$$

$$\tilde{\beta}_0^* = -.0052,$$

$$\tilde{\beta}_1 = \left[6.5808 \quad -10.1692 \quad 7.5255 \quad 6.3\right]', \quad \textit{and}$$

$$\tilde{D} = 1.028.$$

LDA scores for firms are calculated using the function $Z_{i,t} = \tilde{\beta}_0^* + X_{i,t}\tilde{\beta}_1$. Firms with $Z_{i,t}$ scores above a predetermined cutoff point Z_c are classified as healthy and firms with scores below Z_c are classified as failed.

With Logit model, the probability that a firm is healthy is

$$H_{i,t} = \frac{1}{1+\exp\left(-Z_{i,t}\right)}, \quad and \tag{19}$$

$$Z_{i,t} = \gamma_0 + X_{i,t}\gamma_1 \tag{20}$$

where $Z_{i,t}$ is a linear index of financial performance. The above model is estimated using the maximum likelihood method, e.g., Amemiya (1981), p. 1495. The estimated coefficients are:

$$\tilde{\gamma}_0 = 3.2588, \quad and$$

$$\tilde{\gamma}_1 = \left[7.7201 \quad -7.9144 \quad 8.2622 \quad 3.3251\right].$$

All coefficients have the correct signs. Substitution of $Z_{i,t} = \tilde{\gamma}_0 + X_{i,t}\tilde{\gamma}_1$ into $H_{i,t}$ gives the probability of a firm being healthy. Firms with probabilities above a predetermined cut-off probability H_c are classified as healthy and firms with probabilities below H_c are classified as failed.

Jackknife estimates are also computed for the error rates of the above LDA and Logit models. The results are presented in panels B and C of table 2, respectively. A comparison of panels A, B, and C shows that the CUSUM model is superior to both the LDA and Logit models. Specifically, the CUSUM error rates in the healthy and failed samples for w_f = .45 are respectively $P_h = 11.7$ percent and $P_f = 23.94$ percent. These error rates are lower than the respective error rates of LDA and Logit models for all values of w_f. Moreover, the CUSUM model possesses a lower *EC* cost for all values of w_f.

Panels D and E of table 2 present the ratio of expected cost of CUSUM to those of LDA and Logit models, respectively. The results show that the CUSUM model outperforms both the LDA and Logit models in terms of the *EC* criterion. For example, if one were to consider the class of investors who put equal weight on the two types of errors, the cost associated with the use of the CUSUM model would be 73.15 percent that of the LDA model for *s*=1, 88.77 percent for *s*=2, 86.25 percent for *s*=3, and 87.2 percent for *s*=4.

5. Robustness of the CUSUM model

This section addresses the issues of stationarity of the explanatory variables and robustness of the same CUSUM model (e.g., same coefficients and sensitivity parameters) presented in section 4. The robustness issue is explored using the initial sample of 117 healthy firms as well as a new sample of 279 healthy manufacturing firms included in the S&P400 index.

A necessary condition for the four explanatory variables of the CUSUM model to be stationary is that the roots of the polynomial $\det\left(I - \tilde{B}_1 z\right) = 0$ lie outside the complex unit circle, where $\tilde{B}_1$ are the VAR estimates for the four variables from section 4. The roots of the polynomial are 3.5192, 4.7280, 5.9546, and 87.3166. The fact that the roots are greater than one and the means and variances of the variables are bounded (e.g., figures 1 and 2), provide strong support for the hypothesis that all four explanatory variables of the CUSUM model presented in section 4 are stationary.

Figure 4 presents the annual error rates of the CUSUM model for the sample of 117 healthy firms over the period 1978-91. These are calculated using the same model presented in section 4. The straight line represents the model's average error rate for the entire period, which is 17.06 percent. It appears that the CUSUM model exhibits no time trend, thus it is robust over time. The following regression further assesses the stationarity of the model over time:

$$ERR_t = \underset{(.59)*}{.2100} - \underset{(-.05)*}{.0002}\ t, \qquad R^2 = .0002 \tag{20}$$

where ERR_t is the error rate for year t and t = 78,...,91. Note that the error rates are expressed in decimal form and parentheses include the t-values of the estimates. The slope of the regression, $\partial ERR_T/\partial T$, gives the annual growth in the error rates. In the presence of an upward time-trend, the slope of the regression is expected to be positive and statistically significant. Note that the regression slope is close to zero and statistically insignificant at the five-percent level, indicating no time trend. This finding is also supported by the low R-square value of the regression.

Figure 5 presents the annual error rates of the exact CUSUM model over time for the sample of 257 S&P400 firms. The average error rate for the CUSUM model, represented by the straight line on the graph, is 18.84 percent. The regression equation

$$ERR_t = \underset{(.11)*}{.0444} + \underset{(.40)}{.002}\ t, \qquad R^2 = .0129 \tag{21}$$

where ERR_t is the error rate for year t and t = 78,...,91, reaffirms the previous finding that there is no time trend in the error rates, thus the CUSUM model is robust over time.

6. Summary and conclusions

This paper develops a financial distress model for AMEX and NYSE manufacturing and retailing firms using the statistical methodology of time-series Cumulative Sums (CUSUM). Tests show that the model is robust over time and outperforms other models based on the popular statistical methods of Linear Discriminant Analysis (LDA) and Logit.

The model's explanatory variables include the change in the logarithm of deflated total assets, the change in the ratio of fixed assets to total assets, the change in the ratio of operating income to sales, and the change in the ratio of inventory to sales. The first three variables have a positive marginal relationship with the firms' performance scores whereas the change in the ratio of inventory to sales has a negative marginal impact.

Interestingly, none of the popular financial variables included in past financial distress models appears in the CUSUM model as an explanatory variable. Many of these variables exhibit strong positive serial correlation and, in many cases, they are not stationary. The inclusion of such variables in a CUSUM and other statistical models produces financial distress models with deteriorating forecasting performance over time, e.g., Theodossiou and Kahya (1996). Nevertheless, none of these variables or combination of variables produces a better average classification performance than the stationary CUSUM model presented in this paper.

The CUSUM model can be viewed as the dynamic time-series extension of LDA. A desirable feature of the CUSUM model is that it has a very short "memory" with respect to a firm's good performances over the years, but a long "memory" in case of bad performances. The model's memory feature makes it sensitive to negative changes in a firm's financial condition. Consequently, it promptly alerts the financial analyst who may then undertake a closer investigation and assessment of the firm.

The statistical methodology presented in this paper could be applied to other areas such as the rating of corporate or municipal bonds, the assessment of the financial performance of commercial banks and other financial institutions, and the prediction of the debt service problems of countries.

Endnotes

1. A simple measure of serial correlation is given by the autocorrelation function. This is calculated using the formula $\rho(s) = \text{cov}\left(X_{i,t}, X_{i,t-s}\right) / \text{var}\left(X_{i,t}\right)$, where $\text{var}\left(X_{i,t}\right)$ is the variance of $X_{i,t}$ and $\text{cov}\left(X_{i,t}, X_{i,t-s}\right)$ is the covariance between current and past values $X_{i,t}$. For stationary time-series processes $|\rho(s)| < 1$. A multivariate extension of the autocorrelation function can be found in Lütkepohl (1993), pp.25-26.

2. In general, the larger the value of $\rho(s)$, the greater the persistence of deviations of the variables from their means over time and the longer the memory of the process. Note that for non-stationary or random walk time-series processes, $\rho(s) = 1$. In this case, the deviations have infinite persistence and the process has infinite memory, i.e., it never "forgets".

3. The assumption of "homogeneous" unconditional mean of the variables in the healthy group, denoted by $E\left(X_{i,t} \middle| h\right) = \mu_h$, is a basic ingredient in business failure prediction models. The mean μ_h may be viewed as the long-run equilibrium mean value of $X_{i,t}$. In the absence of serial correlation, the conditional mean $E\left(X_{i,t} \middle| h, I_{t-1}\right) = \mu_{h,t-1}$ is equal to μ_h, where I_{t-1} is an information set including past values for the variables.

4. The CUSUM model can be viewed as an extension of the earlier works of Wecker (1979) and Neftci (1982, 1985) on the prediction of turning points of economic time series. Other relevant contributions in this area include Siegmund (1985) and Chu and White (1992).

5. The CUSUM score $C_{i,t}$ behaves as a discrete time continuous random walk process with an upper bound of zero. For healthy firms, the increment (drift) of the process $Z_{i,t} - K$ has a positive mean, provided that $K < D/2$, thus $C_{i,t}$ approaches its upper bound of zero with probability one. As the firm's condition deteriorates, $Z_{i,t} - K$ develops a negative mean and, thereafter, $C_{i,t}$ accumulates negatively, signaling the firms changing condition.

6. For example, a set of 54 variables will generate 7,590,024 four-variables profiles and 379,501,200 five-variables profiles.

7. Note that $N_f = 71$ and not 72 because one of the failing firms has data starting at two years prior to the time of its failure.

8. The jackknife method avoids the problem of bias in the error rates resulting from the model being tested on the same data from which it has been derived. The jackknife method is superior to the holdout method, because it permits the use of all available data in the estimation, resulting in a statistically more reliable model. Also, splitting the data into two or more periods to validate the model over time results in statistically less-reliable estimates for the fitted VAR and CUSUM models. A good review of the various methods used in the estimation of the error rates of linear discriminant analysis and similar models is given in McLachlan (1992), chapter 10, pp. 337-377.

References

Altman, E.I., "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy." *Journal of Finance* 23, pp. 589-609 (September 1968).

Altman, E.I., R.G. Haldeman, and P. Narayanan, "Zeta Analysis, a New Model for Identifying Bankruptcy Risk of Corporation." *Journal of Banking and Finance* 1, pp. 29-54 (June 1977).

Amemiya, T. "Qualitative Response Models: A Survey." *Journal of Economic Literature* 19, pp. 1483-1536 (December 1981).

Beaver, W.H., "Financial Ratios as Predictors of Failure." *Journal of Accounting Research, Supplement*, pp. 71-111 (1966).

Chu, C.J. and H. White, "A Direct Test for Changing Trend." *Journal of Business and Economics Statistics* 10, pp. 289-299 (July 1992).

Dickey, D.A. and W.A. Fuller, "Distribution of the Estimators for Autoregressive Time Series with a Unit Root." *Journal of the American Statistical Association* 74, pp. 427-431 (1979).

Edmister, R.O., "An Empirical Test of Financial Ratio Analysis for Small Business Failure Prediction." *Journal of Financial and Quantitative Analysis* 7, pp.1477-1493 (March 1972).

Gombola, M.J., M.E. Haskins, J.E. Ketz, and D. Williams, "Cash Flow in Bankruptcy Prediction." *Financial Management* 16, pp. 55-65 (Winter 1987).

Johansen, S., *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Advanced Texts in Econometrics, Oxford University Press, New York (1995).

Judge, G.G., W.E. Griffiths, R. Carter Hill, H.L. and T-C. Lee, *The Theory and Practice of Econometrics*. John Wiley, New York (1985).

Lee, C.F., and C. Wu, "Expectation Formation and Financial Ratio Adjustment Processes." *The Accounting Review* 63, pp. 292-306 (1988).

Lo, A.W., "Logit Versus Discriminant Analysis: A Specification Test and Application to Corporate Bankruptcies." *Journal of Econometrics* 31, pp. 151-178 (1986).

Marks, S., and O.J. Dunn, "Discriminant Functions when Covariance Matrices are Unequal." *Journal of the American Statistical Association* 69, pp. 555-559 (1974).

McLachlan, G.J., *Discriminant Analysis and Statistical Pattern Recognition, Chapter 2, Estimation of Error Rates*, Wiley, New York, pp. 337-377 (1992).

Neftci, S.N., "Optimal Prediction of Cyclical Downturns." *Journal of Economic Dynamics and Control* 6, pp. 225-241 (1982).

Neftci, S.N., "A Note on the Use of Local Maxima to Predict Turning Points in Related Series." *Journal of the American Statistical Association* 80, pp. 553-557 (1985).

Ohlson, J.A., "Financial Ratios and the Probabilistic Prediction of Bankruptcy." *Journal of Accounting Research* 18, pp. 109-31 (Spring 1980),.

Siegmund, D., *Sequential Analysis: Tests and Confidence Intervals*. Springer Series in Statistics, Springer-Verlag, New York (1985).

Pastena, V., and W. Ruland, "The Merger/Bankruptcy Alternative." *The Accounting Review* 61, pp. 288-301 (April 1986).

Theodossiou, P., "Predicting Shifts in the Mean of a Multivariate Time Series Process: An Application in Predicting Business Failures." *Journal of the American Statistical Association* 88, pp. 441-449 (June 1993).

Theodossiou, P., and E. Kahya, "Non-Stationarities in Financial Variables and the Prediction of Business Failures." *Proceedings of the Business and Economic Statistics Section, American Statistical Association*, pp.130-133 (1996).

Theodossiou, P., E. Kahya, R. Saidi, and G.C. Philippatos, "Financial Distress Corporate

Acquisitions: Further Empirical Evidence," *Journal of Business Finance and Accounting* 23, pp.699-719 (July 1996).

Wecker, W.E., "Prediction of Turning Points." *Journal of Business* 52, pp. 35-50 (1979).

Appendix

It follows from equations 4 and 5 that $Z_{i,t}$ is equal to:

$$Z_{i,t} = \beta_0 + A_{f,s}\beta_1 + \varepsilon_{i,t}\beta_1 = D/2 + A_{f,s}\beta_1 + \varepsilon_{i,t}\beta_1 .$$

The mean of $Z_{i,t}$ is

$$E\left(Z_{i,t}\right) = D/2 + A_{f,s}\beta_1 .$$

For healthy firms, $A_{f,s} = 0$ and

$$E\left(Z_{i,t}\right) = D/2 .$$

and, for failed firms, using data at one year prior to failure (s =1),

$$E\left(Z_{i,t}\right) = D/2 + A_f\beta_1 = D/2 - (1/D)A_f\Sigma^{-1}A_f'$$

$$= D/2 - D = -D/2$$

Moreover, because the residuals are uncorrelated over time, individual $Z_{i,t}$ scores for healthy and failed firms are expected to deviate randomly over time around their population means. The variance of $Z_{i,t}$ for healthy and failed firms is

$$\operatorname{var}\left(Z_{i,t}\right) = E\left(\beta_1'\varepsilon_{i,t}'\varepsilon_{i,t}\beta_1\right) = \beta_1'\Sigma\beta_1$$

$$= \left(1/D^2\right)A_f\Sigma^{-1}\Sigma\Sigma^{-1}A_f' = \left(1/D^2\right)D^2 = 1.$$

Table 1. Financial variables considered.

Variables	Computation	Proxy	Used in
Cash to current liabilities	V1/V5	Liquidity	Beaver (1966), Edmister (1972), Gombola et al. (1987)
Cash to total assets	V1/V6	Liquidity	Beaver (1966), Gombola et al. (1987)
Current assets to current liabilities	V4/V5	Liquidity	Beaver (1966), Altman et al. (1977), Gombola et al. (1987)
Current assets to total assets	V4/V6	Liquidity	Beaver (1966), Lo (1986), Gombola et al. (1987)
Net working capital to total assets	(V4–V5)/V6	Liquidity	Beaver (1966), Altman (1968), Ohlson (1980), Theodossiou (1993)
Net working capital to sales	(V4–V5)/V12	Liquidity	Edmister (1972)
Quick assets to current liabilities	(V4–V3)/V5	Liquidity	Beaver (1966)
Gross profit to sales	(V12–V41)/V12	Profitability	
Net income book value of equity	V172/V216	Profitability	
Net income to fixed assets	V172/V8	Profitability	
Net income to total assets	V172/V6	Profitability	Beaver (1966), Ohlson (1980), Lo (1986), Gombola et al. (1987)
Operating income to fixed assets	V13/V8	Profitability	
Operating income to sales	V13/V12	Profitability	Theodossiou et al. (1996)
Operating income to total assets	V13/V6	Profitability	Altman (1968), Altman et al. (1977)*, Theodossiou (1993)
Retained earnings to total assets	V36/V6	Long-term Profitability	Altman (1968), Altman et al. (1977)

Table 1. (continued)

Variables	Computation	Proxy	Used in
Long-term debt to total assets	V9/V6	Financial Leverage	Beaver (1966), Altman (1968)
Total Liabilities to total assets	V181/V6	Financial Leverage	Ohlson (1980), Gombola et al. (1987), Theodossiou et al. (1996)
MVE to total liabilities	(V24*V25)/V181	Market Structure	Altman (1968)
Logarithm of deflated fixed assets	log(100*(V8/PPI))	Size	
Logarithm of deflated sales	log(100*(V12/PPI))	Size	Pastena and Ruland (1986)
Logarithm of deflated total assets	log(100*(V6/PPI))	Size	Altman et al. (1977), Ohlson (1980), Lo (1986), Theodossiou et al. (1996)
Logarithm of number of employees	log(V29)	Size	
Accounts receivable to current assets	V2/V4	Management Efficiency	
Accounts receivable to sales	V2/V12	Management Efficiency	Beaver (1966), Gombola et al. (1987)**
Fixed assets to total assets	V8/V6	Operating Leverage	Theodossiou (1993)
Inventory to sales	V3/V12	Management Efficiency	Beaver (1966), Edmister (1972), Theodossiou (1993), Theodossiou et al. (1996)
Sales to total assets	V12/V6	Activity	Altman (1968), Gombola et al. (1987)

Notes: This paper also considers the annual changes in the values of the above variables from year t–1 to year t. The citations indicate studies that considered the variables. *Altman (1968) and Altman et al. (1977) used the ratio of EBIT (Earnings Before Interest and Taxes) to total assets; ** Gombola et al. (1987) used the reciprocal of the ratio of accounts receivable to sales. The numbers following the letter "V" are the numbers assigned to the variables in the COMPUSTAT manual. PPI is the producer price index.

Table 2. Error rates for the CUSUM, LDA and Logit models

A. Optimal values of K and L and error rates for the CUSUM model

w_f	K	L	EC	P_h	P_f	$P_{f,2}$	$P_{f,3}$	$P_{f,4}$
.4	.0939	1.7601	16.5	6.84	30.99	55.56	66.67	71.83
.45	.1056	1.2907	17.21	11.7	23.94	48.61	54.17	63.38
.5	.0587	.8214	17.69	17.06	18.31	40.28	45.83	60.56
.55	.0117	.5867	17.65	20.28	15.49	37.5	37.5	53.52
.6	.0821	.704	17.25	22.01	14.08	37.5	37.5	53.52

B. Optimal cut-off points Z_c and error rates for the LDA model

w_f	Z_c	EC	P_h	P_f	$P_{f,2}$	$P_{f,3}$	$P_{f,4}$
.4	–.2288	22.67	16.19	32.39	47.22	55.56	71.83
.45	–.2288	23.48	16.19	32.39	47.22	55.56	71.83
.5	–.2053	24.18	17.36	30.99	47.22	55.56	71.83
.55	–.0997	24.58	21.91	26.76	45.83	51.39	63.38
.6	–.0997	24.82	21.91	26.76	45.83	51.39	63.38

C. Optimal cut-off points H_c and error rates for the Logit model

w_f	H_c	EC	P_h	P_f	$P_{f,2}$	$P_{f,3}$	$P_{f,4}$
.4	.9463	22.09	12.41	36.62	51.39	59.72	76.06
.45	.9522	23.28	15.83	32.39	51.39	56.94	70.42
.5	.9522	24.11	15.83	32.39	51.39	56.94	70.42
.55	.9638	24.39	26.66	22.54	41.67	44.44	57.75
.6	.9638	24.19	26.66	22.54	41.67	44.44	57.75

D. Ratio of CUSUM to LDA expected cost

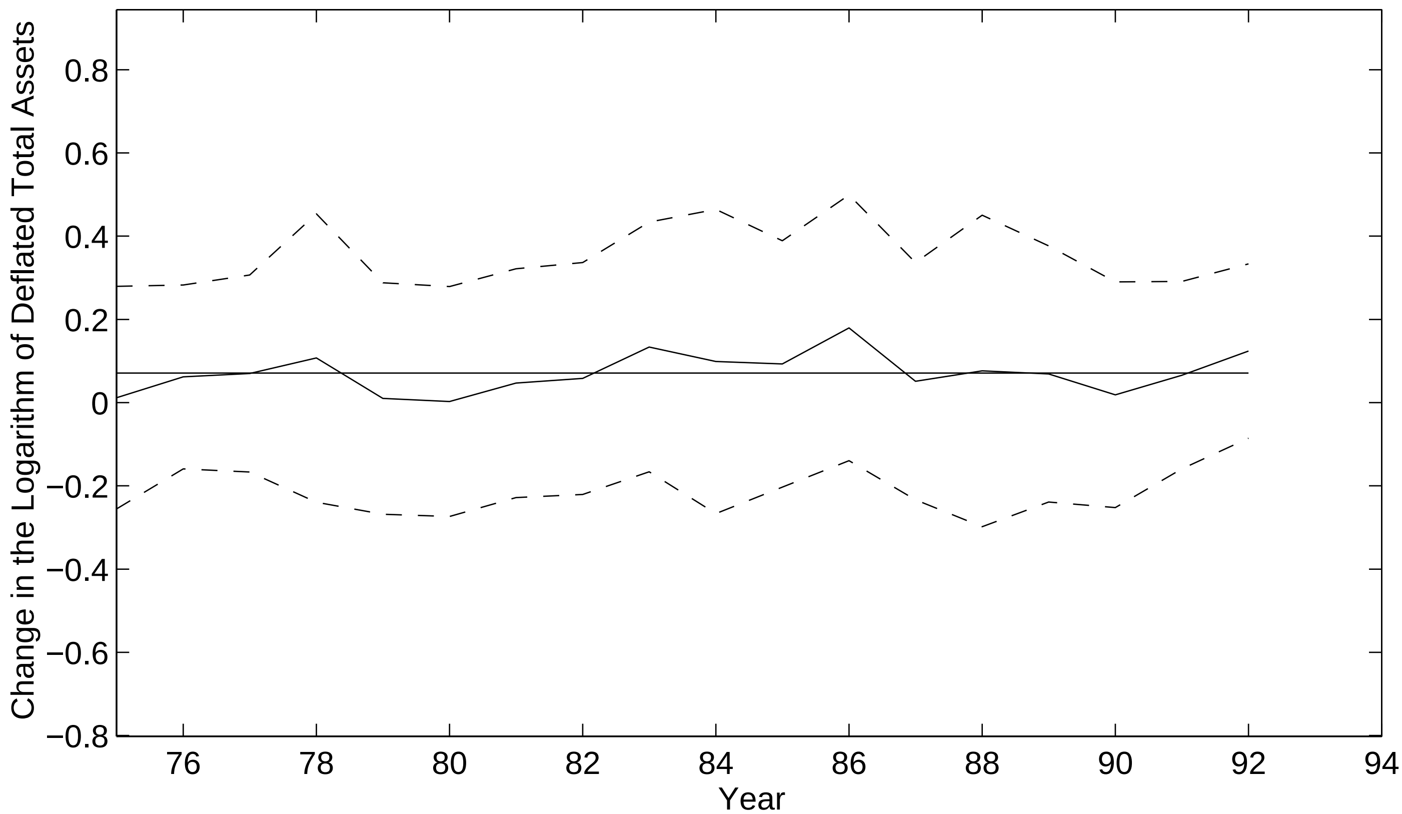
w_f	s=1	s=2	s=3	s=4
.4	72.78	92.05	96.36	85.41
.45	73.28	93.88	90.87	84.78
.5	73.15	88.77	86.25	87.02
.55	71.79	84.83	78.03	86.23
.6	69.52	86.32	79.06	87.45

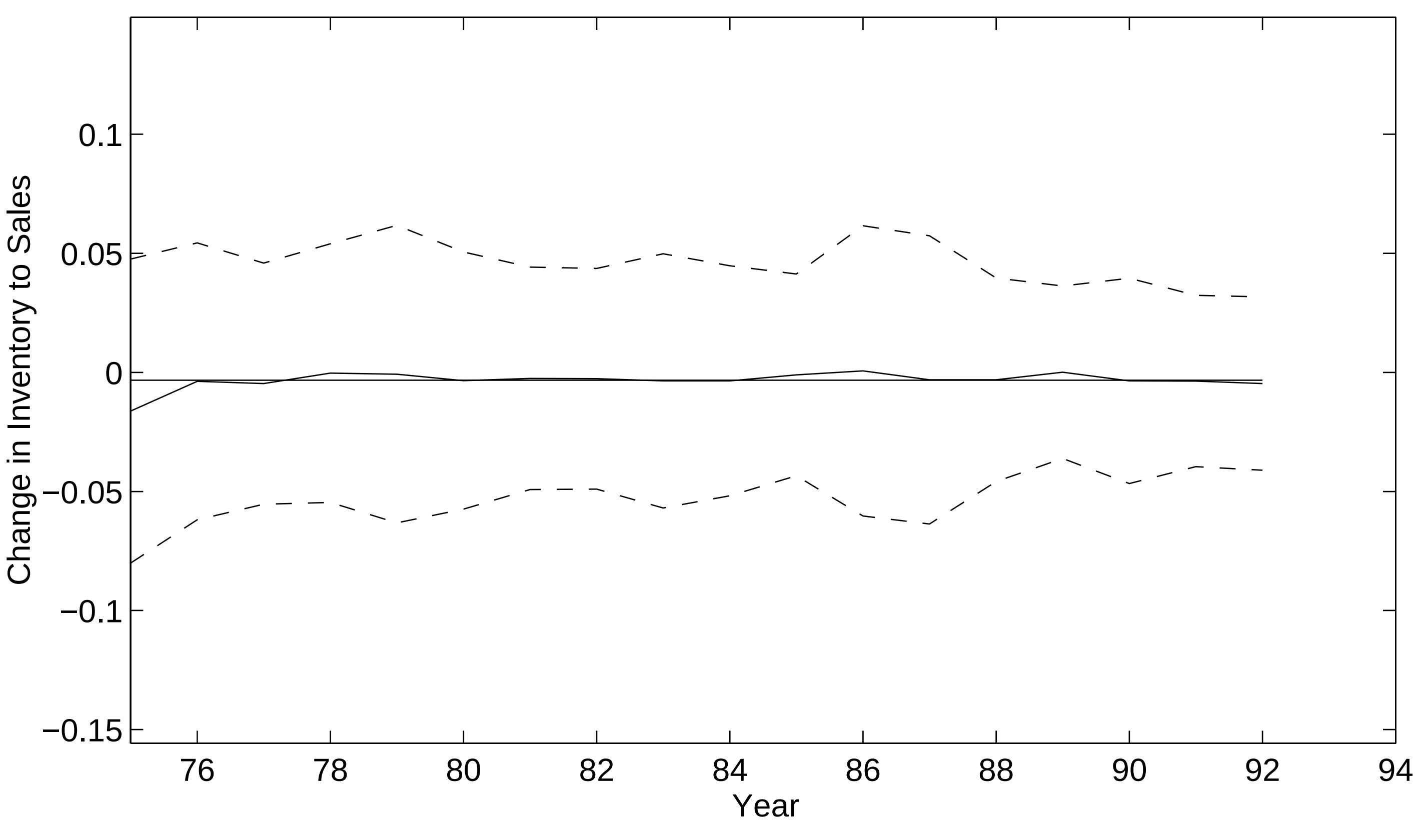
Table 2. (continued)

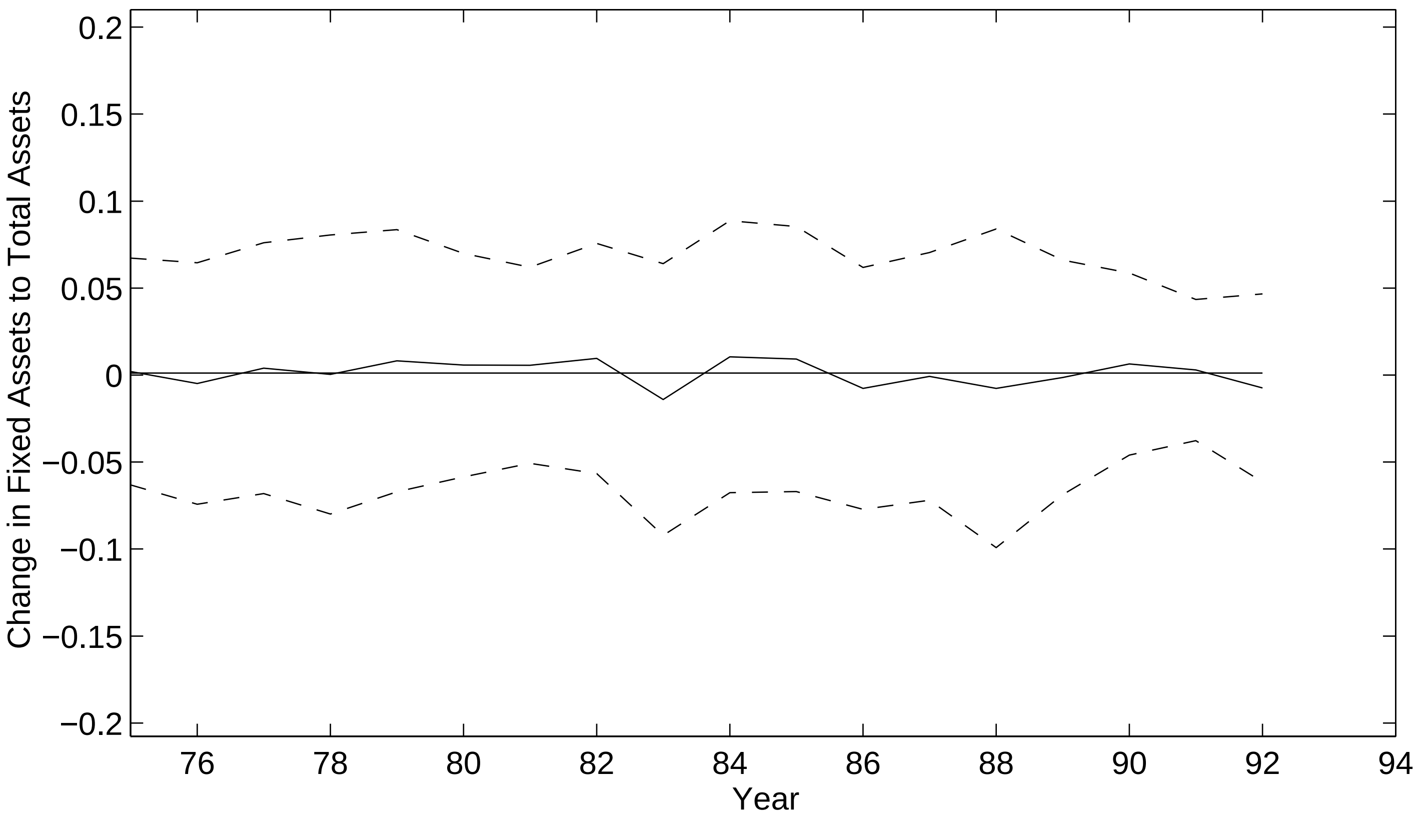
	E. Ratio of CUSUM to Logit expected cost			
w_f	s=1	s=2	s=3	s=4
.4	74.68	94.02	98.21	86.72
.45	73.90	88.93	89.73	86.52
<u>.5</u>	<u>73.34</u>	<u>85.29</u>	<u>86.42</u>	<u>89.99</u>
.55	72.34	85.21	81.64	88.12
.6	71.35	87.78	83.86	90.3

Notes: P_h is the percentage of healthy firms misclassified by the models. $P_f = P_{f,1}$ is the percentage of failed firms misclassified by the models using data from one year prior to the point of failure. $P_{f,2}$, $P_{f,3}$ and $P_{f,4}$ are respectively the percentages of failed firms misclassified by the models using data two, three and four years prior to the time of failure. As expected, these error rates increase because it is more difficult to predict financial distress further back from the point of failure. The expected cost function for each model is $EC_s = w_f P_{f,s} + (1 - w_f)P_h$, for s = 1, 2, 3 and 4. Note that by definition, $EC \equiv EC_1$. The values for K and L, Z_c and H_c are those that minimize the EC function of the CUSUM, LDA and Logit models for each w_f.

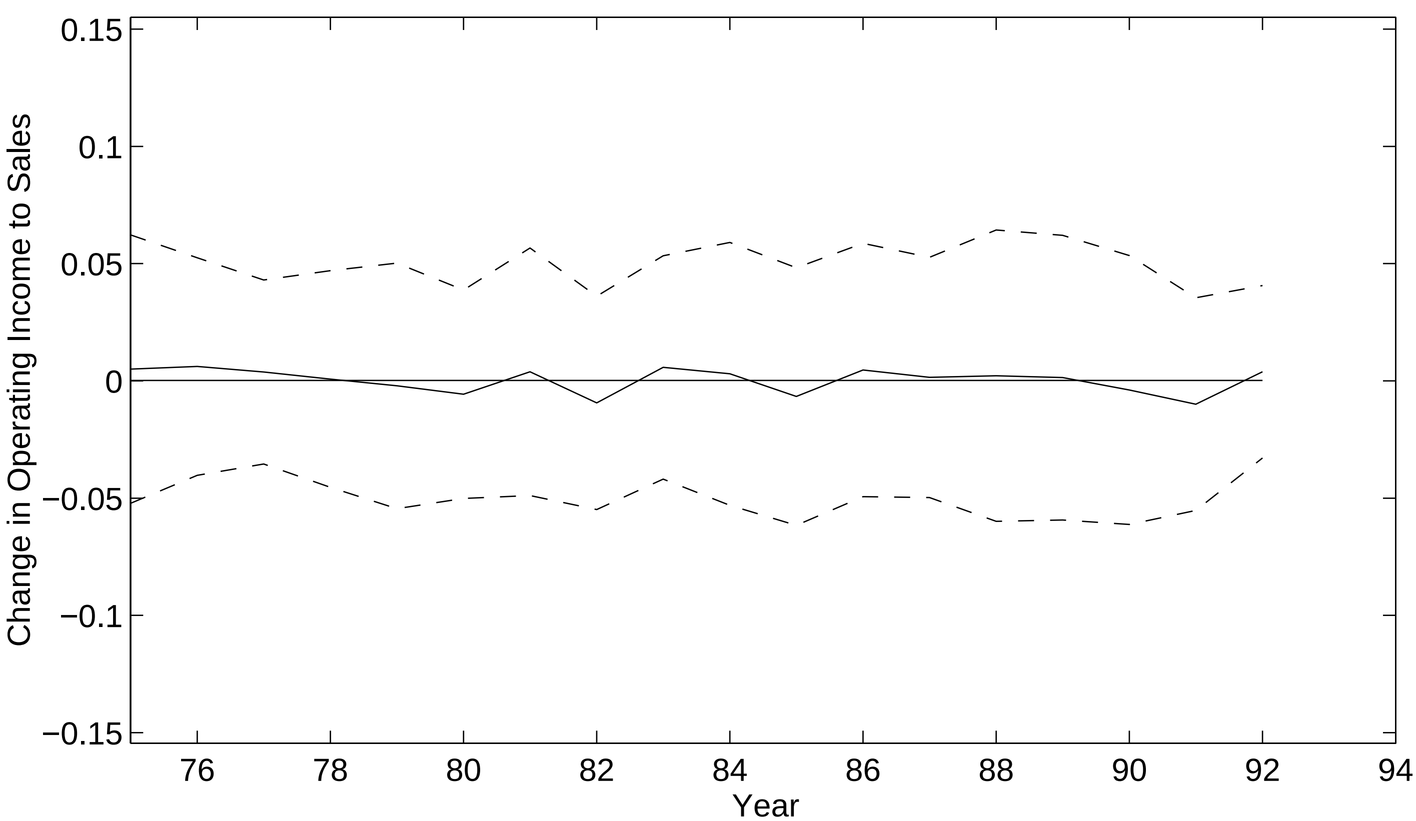
Figure 1. Means and Standard Deviations of the Variables in the Healthy Sample





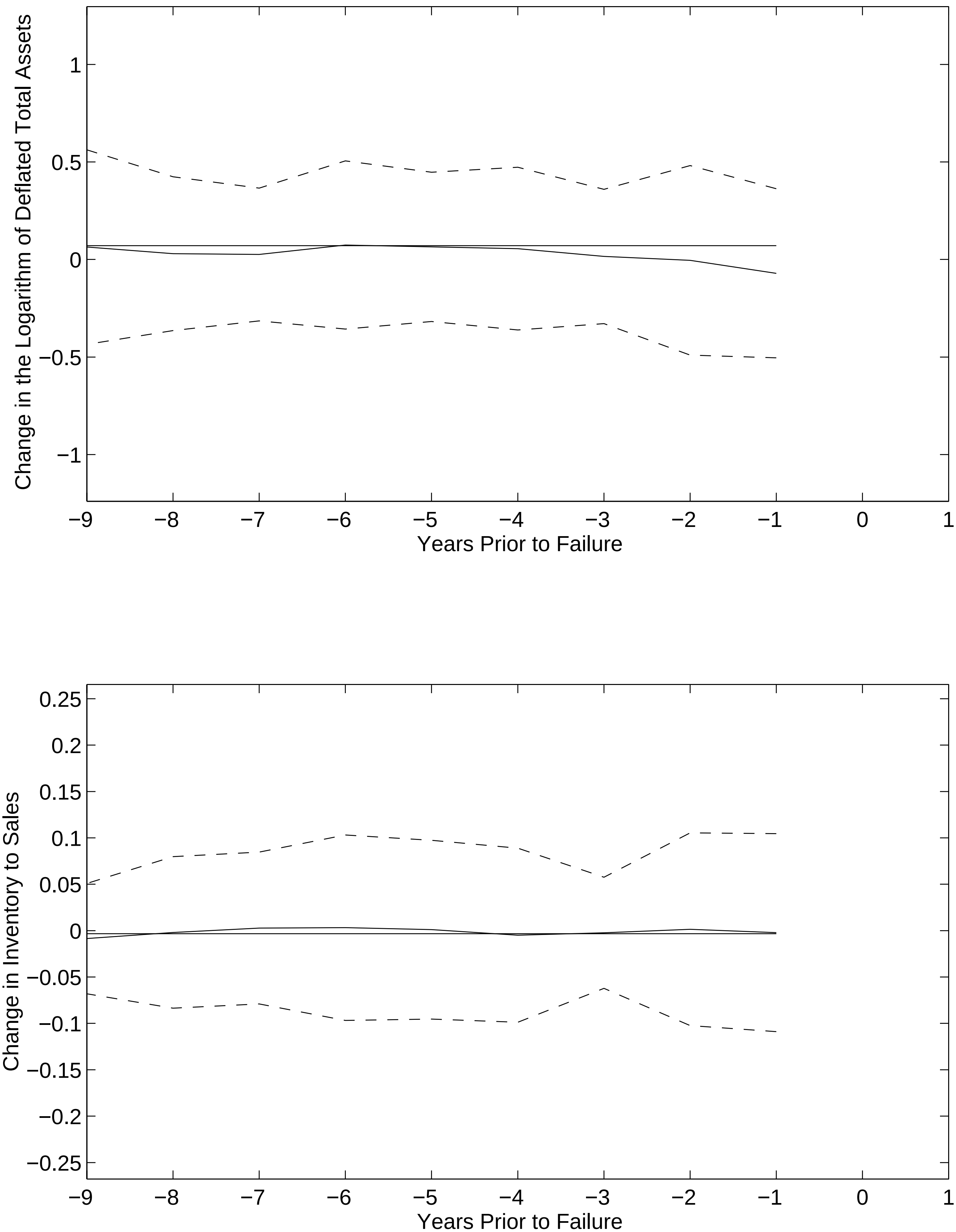


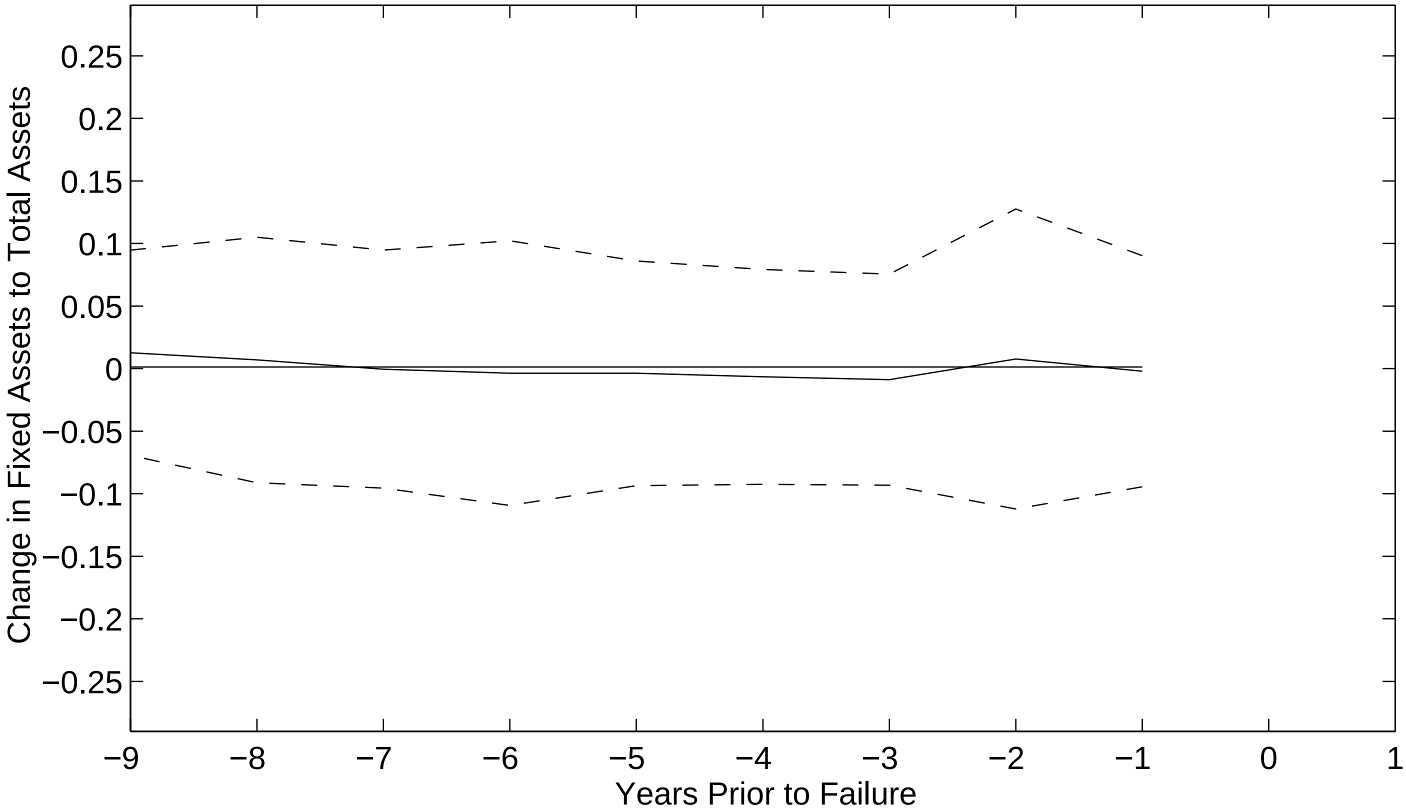
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
76
78
80
82
84
86
88
90
92
94
Year
Change in Fixed Assets to Total Assets



0.15
0.1
0.05
0
−0.05
−0.1
−0.15
76
78
80
82
84
86
88
90
92
94
Year
Change in Operating Income to Sales

Figure 2. Means and Standard Deviations of the Variables in the Failed Sample




Change in Fixed Assets to Total Assets
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
−9
−8
−7
−6
−5
−4
−3
−2
−1
0
1
Years Prior to Failure

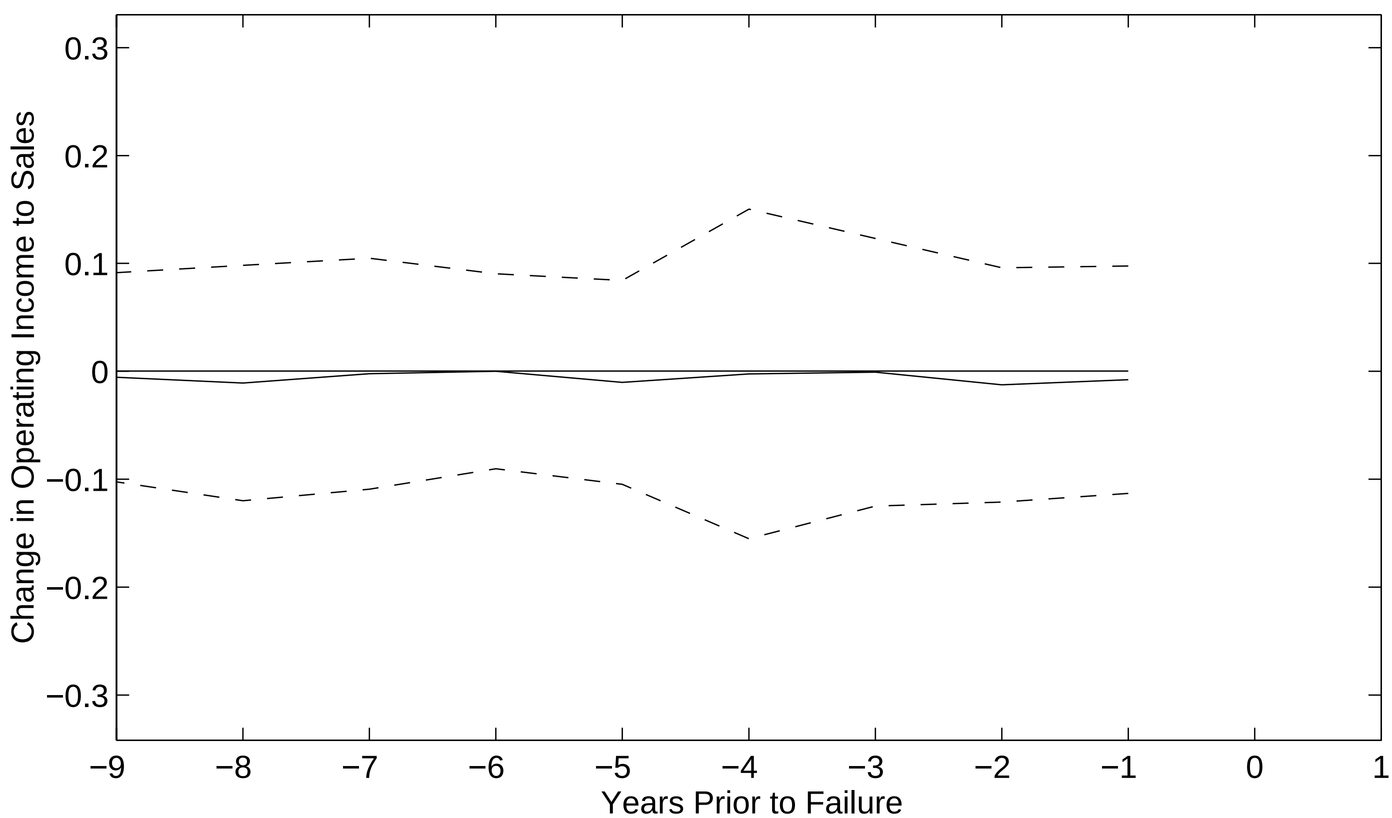

Change in Operating Income to Sales
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−9
−8
−7
−6
−5
−4
−3
−2
−1
0
1
Years Prior to Failure

Figure 3. Means of the Z−Scores for Failed Firms

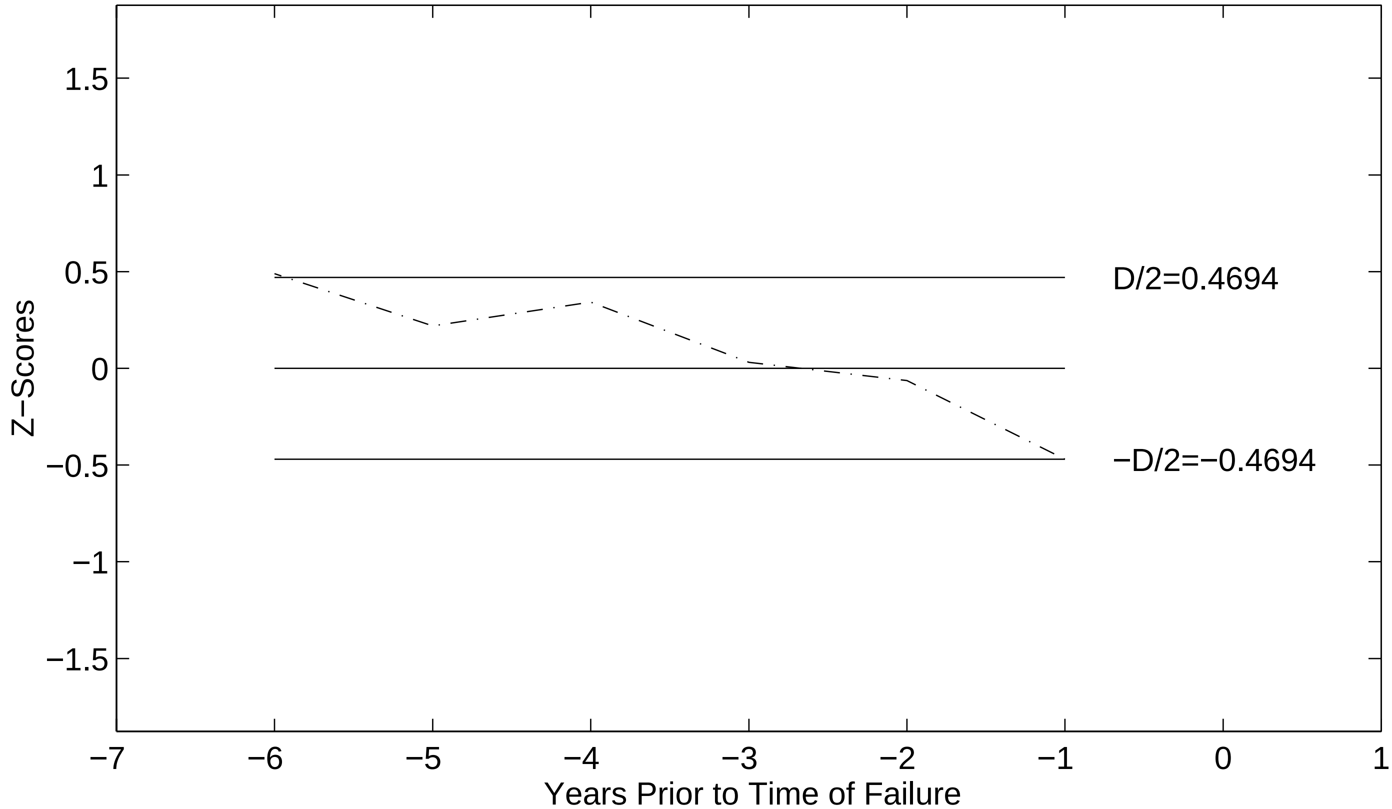


Figure 4. Annual Error Rates of the CUSUM Model for Healthy Firms

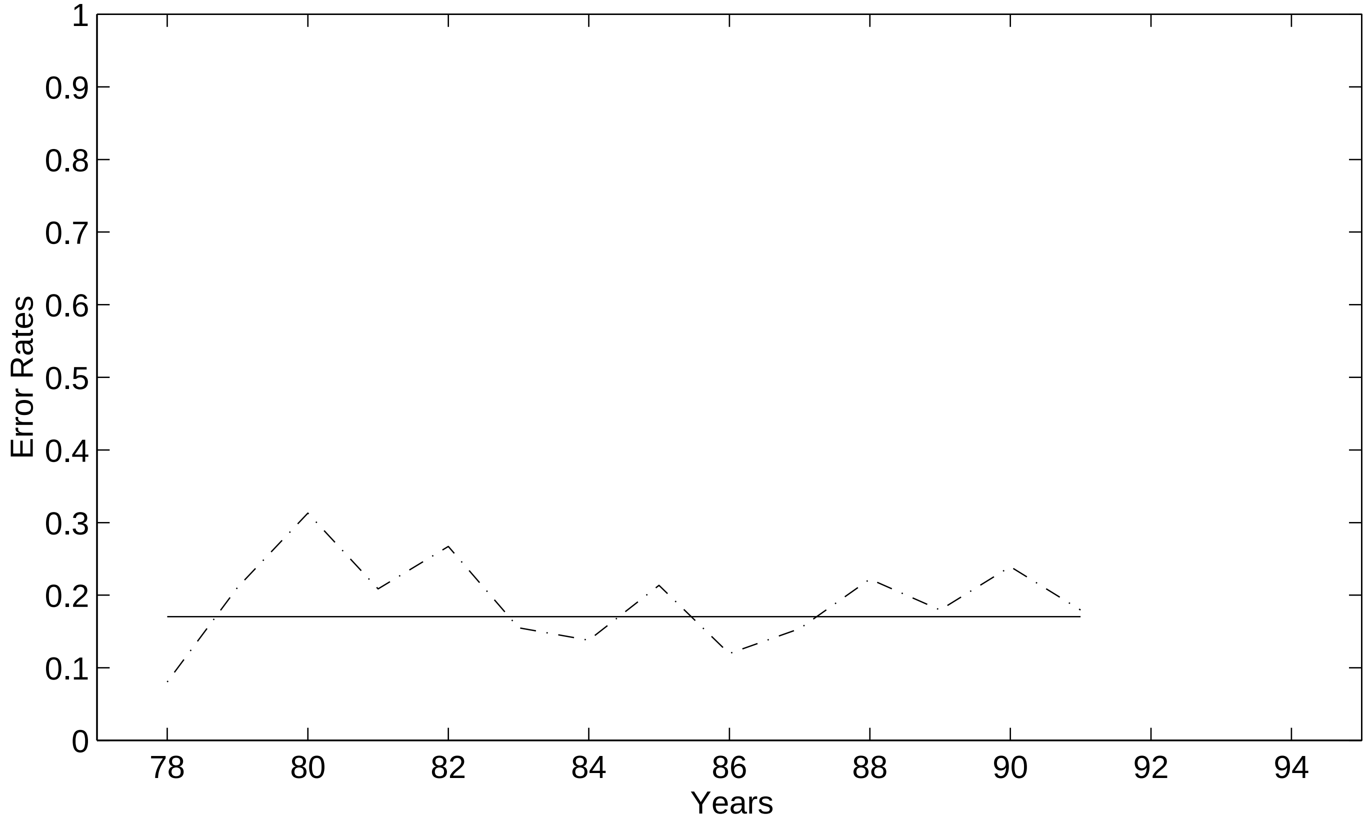


Figure 5. Annual Error Rates of the CUSUM Model for the S&P400 Firms